With the improved design there is no need to control the total operation time \( t_0 \), while the time dependence of the voltage and flux can be optimized such that the time span of the manipulations is long enough to simplify time control and short enough to speed up the computation.

Also, the circuit of the current source, with resistance \( R_J \), which couples the flux \( \Phi_1 \) to the SQUID by the mutual inductance \( M \), introduces fluctuations and may destroy the coherence of the qubit dynamics. At the degeneracy point, the decoherence time is \( \tau_1 \) \( \approx \frac{1}{\pi} \frac{1}{R_J} \left| \langle \Phi_1 (E^\prime) M \rangle \right|^2 / (\hbar k_B T) \). This dephasing is slow if the current source is coupled weakly to the qubit (small \( M \)) and its resistance is high.

The control of the Josephson energies \( E_\ell (\Phi_1) \) provides the possibility of coupling each selected pair of qubits, while keeping all the other ones uncoupled, bringing us close to the ideal model of equation (1). The simplest implementation of the coupling is to connect all \( N \) qubits in parallel with each other, and with inductor \( L \) (Fig. 3). Fast oscillations in the resulting LC-circuit produce an effective coupling of the qubits

\[
H_{int} = - \sum_{\ell} \frac{E_\ell (\Phi_1) E_\ell (\Phi_2)}{E_L} \hat{\sigma}_x \hat{\sigma}_x
\]

where \( E_\ell = \frac{\Phi_\ell (L \pi L)}{(C_\ell / C_{q_\ell})^2} \). The coupling shown in equation (6) can be understood as the magnetic energy of the inductor which is biased by a current composed of contributions from all qubits, \( \Phi_\ell \approx \Phi_\ell (E_\ell \pi E_L) \).

With this design we can perform all gate operations. In the idle state the interaction hamiltonian of equation (6) is zero as all the Josephson couplings are turned off. The same is true during a \( \Phi_\ell \)-evolution of the density matrix of the coupled system.

To demonstrate that the constraints on the set of system parameters can be met by available technology, we suggest a suitable set. We choose junctions with capacitance \( C_\ell = 300 aF \), corresponding to a charging energy (in temperature units) \( E_c \approx 3K \), and a smaller gate capacitance \( C = 30 aF \) to reduce the coupling to the environment (even lower \( C \) are available and improve the performance further). The superconducting gap has to be slightly larger, \( \Delta > E_c \). Thus at a working temperature of the order of \( T = 50 mK \), the initial thermalization is assured. We further choose \( E_\ell = 50 mK \); so the timescale of one-qubit operations is \( \tau_{op} \approx h / E_\ell \approx 70 ps \). Fluctuations associated with the gate voltages (equation (4)), with resistance \( R_G \approx 50 \Omega \), limit the coherence time to \( \tau_1 / \tau_{op} \approx 4000 \) operations. With the parameters of the flux-circuit \( L_\ell = 0.1 nH \), \( M = 1 nH \) and \( R_L \approx 10^{10} \Omega \), current fluctuations have a weak dephasing effect. To assure fast two-bit operations, we choose the energy scale \( E_L \) to be of the order of \( 10 E_c \), which is achieved for \( L \approx 3 \mu H \). With these parameters, the number of qubits in the circuit can be chosen in the range of 10–50, of course at the expense of shorter coherence times \( \tau_1 / N \).

Some further remarks are in order.

(1) After the gate operations, the resulting quantum state has to be read out. This can be achieved by coupling a normal-state single-electron transistor capacitively to a qubit. The important aspect is that during computation the transistor is kept in a zero-current state and adds only to the total capacitance. When the transport voltage is turned on, the phase coherence of the qubit is destroyed, and the dissipative current in the transistor, which depends on the state of the qubit, can be read out. This quantum measurement process has been described explicitly in ref. 16 by an analysis of the time-evolution of the density matrix of the coupled system.

(2) Inaccuracy in the control of fluxes, voltages and the time-span of operations leads to diffusion of the actual quantum state from the one that exists in the absence of errors. A random error of order \( \epsilon \) per gate limits the number of operations to a value which is of order \( \epsilon^{-2} \). For the circuit parameters above, \( \epsilon = 1\% \) would lead to smaller effects than those produced by environment.

(3) Many powerful quantum algorithms make use of parallel operations on different qubits. Although this is not possible with the present system, it may be achievable by a more advanced design, making use of further tunable SQUIDs decoupling different parts of the circuit. Such modifications, as well as the further progress of nanotechnology, should provide longer coherence times and allow scaling to larger numbers of qubits.

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Evidence against ‘ultrahard’ thermal turbulence at very high Rayleigh numbers

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Several theories1–3 predict that a limiting and universal turbulent regime—‘ultrahard’ turbulence—should occur at large Rayleigh numbers (Ra, the ratio between thermal driving and viscous dissipative forces) in Rayleigh–Bénard thermal convection in a closed, rigid-walled cell. In this regime, viscosity becomes negligible, gravitationally driven buoyant plumes transport the heat

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and the thermal boundary layer, where the temperature profile is linear, controls the rate of thermal transport. The ultrahard state is predicted to support more efficient thermal transport than ‘hard’ (fully developed) turbulence: transport efficiency in the ultrahard state grows as \( Ra^{1/2} \), as opposed to \( Ra^{2/7} \) in the hard state. The detection of a transition to the ultrahard state has been claimed in recent experiments using mercury and gaseous helium. Here we report experiments on Rayleigh–Bénard convection in mercury at high effective Rayleigh numbers, in which we see no evidence of a transition to an ultrahard state. Our results suggest that the limiting state of thermal turbulence at high Rayleigh numbers is ordinary hard turbulence.

In thermal turbulence, several dimensionless numbers determine the degree of turbulence. The most important of these is the Rayleigh number, \( Ra \), the ratio between thermal driving and viscous dissipation forces: \( Ra = \alpha g \Delta T l / k \nu \), where \( \alpha \) is the coefficient of thermal expansion, \( \kappa \) the thermal conductivity, \( \nu \) the kinematic viscosity, \( g \) the gravitational acceleration, \( l \) the typical length and \( \Delta T \) the temperature difference across the cell. The second important quantity is the dimensionless velocity, the Reynolds number, \( Re \), the ratio of shear forces to viscous forces: \( Re = \nu l / \nu \), where \( \nu \) is a typical velocity. The fluid’s Prandtl number, \( Pr \), is the ratio between thermal and viscous dissipation: \( Pr = \nu / \kappa \). Typical gases have \( Pr \approx 1 \), whereas \( Pr > 1 \) for most liquids such as water and oil. The actual flow patterns of the fluid can be described in an averaged sense by the Nusselt number, \( Nu \), the thermal transport efficiency, which is the ratio between gross thermal transport (including advection and diffusion) and diffusive thermal transport in the absence of flow. The larger \( Nu \), the more efficient the convective thermal transport.

Convective thermal turbulence is inevitably anisotropic and non-homogeneous at large length scales. Two types of boundary layers are important. Near the container walls, flow velocity vanishes due to the no-slip condition which creates a highly sheared viscous boundary layer. The temperature profile becomes linear near the top and bottom walls, because only diffusion transports heat where advection is suppressed. The time-averaged temperature in the bulk is constant and equal to the average of the top and bottom plate temperatures due to strong mixing by the turbulent flow. The time-averaged profile is steep and linear in the thermal boundary layer. The thickness of the thermal boundary layer, which is the inverse of the temperature gradient, limits the gross heat transport across the turbulent cell. Thus \( Nu \) is proportional to the ratio between the cell height and the thickness of the thermal boundary layer.

The theories predicting \( Nu \propto Ra^{1/2} \) make one of two assumptions. The first is that, because the thickness of the viscous boundary layer varies as \( \sim Ra^{-1/2} \), it becomes negligible at very high \( Ra \). Thus heat is advected by buoyant structures (for instance, plumes or thermals, rising masses of hot fluid or falling masses of cold fluid of classical mushroom-cloud shape) which move at the free fall velocity. That is, as viscous forces are negligible compared with inertial forces in this regime, the thermals accelerate as if they were free, undamped probes.

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Temperature probes

Cioni et al. proposed modifications to the theory which predicted \( Ra_{crit} = 5 \times 10^5 \) in He. The Libchaber experiments on turbulence in Rayleigh–Bénard convection in low temperature gaseous helium found no indication of a shift in \( Nu \) scaling up to \( Ra \approx 10^{14} \), but did see a suggestive change in the temperature power spectrum.

Because \( Pr \approx 0.025 \) at 20°C in Hg compared to \( Pr = 0.7 \) for He gas below 1 atm at 5 K, searching for a new range in Hg required only a \( Ra \) number of the order of \( 10^7 \). Our initial Hg experiments used cylindrical cells, with aspect ratios of one-half, one, and two, to reach \( Ra \approx 2 \times 10^5 \). Our results were compatible with the hard turbulence \( Nu \)-versus-\( Ra \) scaling exponent of \( 2/7 \). The break in the data for an aspect ratio of two at \( Ra \approx 2 \times 10^5 \) results from a pattern competition instability in the flow. Small \( Ra \approx 10^5 \), strong steady bulk mean flow in the cell of aspect ratio one reduces the measured scaling exponent to 0.25 (ref. 17).

The exponent increases towards \( 2/7 \) with increasing \( Ra \).

These measurements appear to invalidate both of the mechanisms proposed for the onset of ultrahard turbulence. The thermal and viscous boundary layers cross below \( Ra \approx 10^5 \), so we should definitely have seen the transition if the boundary crossing theory was correct. The boundary layer thickness argument also seems to fail: in Hg, after the onset of hard turbulence and when the viscous boundary layer crosses the thermal (above \( Ra \approx 10^5 \)), our experiments have shown that both boundary layers shrink together with a nearly constant ratio and a scaling exponent of \( ~0.20 \pm 0.02 \), different from the theoretical value of \( ~1/2 \) (ref. 17).

Cioni et al., using a larger Hg cell of aspect ratio one, measured up to \( Ra \approx 5 \times 10^5 \). They claimed that the \( Nu \)-versus-\( Ra \) scaling exponent increased above \( 2/7 \) at a single point at the very top of their \( Ra \) range, but could not measure the new value. Chavanne et al. studied Rayleigh–Bénard convection with an aspect ratio of one-half in gaseous and liquid He. In gas near the gas–liquid critical point they saw an increasing \( Nu \)-versus-\( Ra \) scaling exponent above \( Ra \approx 10^{11} \) and measured an approximate power law of

\[ Nu \propto Ra^{1/2} \]
In Fig. 2 we show the Nu-versus-Ra curve for the combined data from our four experiments. From $Ra = 2 \times 10^5$ to $8 \times 10^{10}$, the Nusselt numbers lie on top of each other with a constant scaling exponent of $0.29 \pm 0.01$. This Rayleigh number is, to our knowledge, the highest yet achieved in thermal convection in a fluid of low Prandtl number, and the estimated Reynolds number is $5 \times 10^9$ (ref. 17), which is higher than that of Chavanne et al. Even at the highest Ra, our data show no indication of an increase in power law.

The Nu-versus-Ra curve is a highly averaged characterization of the fluid flow. In particular, transitions which change the temperature histograms and power spectra (which have never been observed) might not change the Nu-versus-Ra exponent. Figure 3 compares the power spectra and histograms for temperature time series taken for $Ra = 1.85 \times 10^8$ and $Ra = 5.14 \times 10^{10}$. The best fit to the scaling range of the temperature power-spectrum gives a slope of $-1.47 \pm 0.07$. This slope and the range (less than five decades) is compatible with the result of Wu et al. obtained for convective hard turbulence in helium. Except for the expected increase in inertial scaling range and the shrinking width of exponential decay in the temperature histogram, the two curves correspond exactly to each other. The small asymmetry in the histogram shape results from the location of the temperature probe above the centre of the cell, 15 cm from the cold top plate.

Again we see no sign of a qualitative change in the turbulence.

Chavanne et al. clearly see an interesting effect in gaseous He—an increase of Nu at very high Ra—but the transition is gradual and never shows a clear power law, certainly not $Ra^{1/2}$. One complication in interpreting their results is that Pr changes rapidly near the critical point so that it is not constant from the top to the bottom of their cell. Although it is tempting to attribute this regime to ultrahard turbulence, it may instead be due to Pr effects, which partially mimic the predicted behaviour of ultrahard turbulence.

For our largest Ra, the Pr at the top plate (at $T = 130^\circ$C) is 0.018 and at the bottom plate Pr = 0.024 at $T = 42^\circ$C, a change of 35%. However the maximum Pr is 0.027, so viscous damping is always...
more difficult to observe. Time-resolved X-ray diffraction has the potential to probe fast, atomic-scale motions \(^3\). This is made possible by the generation of ultrashort X-ray pulses \(^5\)-\(^8\), and several X-ray studies of fast dynamics have been reported \(^9\)-\(^13\). Here we report the direct observation of coherent acoustic phonon propagation in crystalline gallium arsenide using a non-thermal, ultrafast-laser-driven plasma—a high-brightness, laboratory-scale source of subpicosecond X-ray pulses \(^6\)-\(^15\). We are able to follow a 100-ps coherent acoustic pulse, generated through optical excitation of the crystal surface, as it propagates through the X-ray penetration depth. The time-resolved diffraction data are in excellent agreement with theoretical predictions for coherent phonon excitation \(^16\) in solids, demonstrating that it is possible to obtain quantitative information on atomic motions in bulk media during picosecond-scale lattice dynamics.

Crystalline gallium arsenide (GaAs) is available in large samples of very high crystalline quality, making it an ideal system for quantitative investigations using optical pumping and X-ray probing. Its physical parameters are known with great precision \(^1\), and numerous prior studies \(^1\) on its ultrafast electronic properties provide a solid foundation for interpretation, and for testing the potential of ultrafast X-ray diffraction. Indeed, some information on ultrafast lattice dynamics after optical excitation has already been indirectly inferred from a variety of linear and nonlinear optical techniques \(^2\)-\(^4\). But owing to the short penetration depth of visible light, information on bulk dynamics in absorbing materials has not yet been obtained.

Figure 1 shows a schematic of our experiment. An ultrashort pulse of laser-generated Cu K\(_\alpha\) X-rays (consisting of two closely spaced lines, K\(_\alpha_1\) and K\(_\alpha_2\)) diffracts in a symmetric Bragg configuration from the 3.26-Å (111) lattice spacing in GaAs, penetrating \(\sim 2\) μm into the bulk through the surface normal. A 30-fs pump pulse with variable time delay generates electron–hole pairs via interband excitation within the submicrometre penetration depth of the 800-nm light. By illuminating with light only a portion of the area probed by X-rays, we simultaneously observe the K\(_\alpha\) lines from both photopumped and unperturbed areas of the semiconductor surface. Two-minute exposure X-ray–CCD images are normalized with respect to the incident X-ray flux and binned within the region of uniform illumination. In Fig. 2, the measured and calculated (described below) diffraction profiles are shown as a function of angular deviation from the Bragg angle, \(\theta - \theta_0\), and of pump–probe time delay. Zero delay corresponds to the initial deviation of the diffracted signal. We observe for these early times that both of the original K\(_\alpha\) lines broaden and shift slightly to larger angles (\(\sim 20\) arcsec), while two new lines appear, broader and weaker than the original lines and deviate by approximately \(\sim 150\) arcsec relative to the original Bragg angle. As the pump–probe delay is increased, these new lines decrease in width, increase in intensity, and merge asymptotically with the still broadened and shifted main lines. Finally, the angle-integrated diffraction signal (not shown) increases monotonically with delay, reaching a plateau of twice the unperturbed value.

The incident optical energy couples into the material by promoting electrons from the valence to the conduction band. Single- and two-photon (as well as free-carrier) absorption contribute to excitation during absorption of the pump pulse. Bandgap renormalization and state filling, not well characterized at these fluences, introduce additional nonlinearities to the excitation process. Also, efficient carrier diffusion at early times may smooth the energy deposition profile in a few picoseconds. After absorption, energy is transferred to the lattice via intraband relaxation \(^7\) and delayed Auger heating \(^8\). Existing estimates of the Auger coefficient \(^9\) in GaAs indicate that most of the energy is efficiently transferred to the lattice within a few picoseconds. X-ray diffraction, primarily sensitive to the acoustic phonon population, occurs only after the decay of the initially generated longitudinal optical phonons \(^10\). Following